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NAVSPASUR Sensor Performance Study

by

Stephen F. Schaaf

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The Naval Space Surveillance Command (NAVSPASUR) has been in existence since the late 1950's. Operating with a formidable array of three large transmitters and six receiving stations, the command has carried out the mission of surveilling and cataloging all space objects in near earth orbit. To date, over 21,000 artificial satellites have been tracked and catalogued by the command. In order to document how effectively the fence has been accomplishing this mission, this thesis has been undertaken to provide NAVSPASUR with a statistics based measure of demonstrated system detection performance.

It is the purpose of this thesis to provide NAVSPASUR with a scientific study and evaluation of system performance and capabilities as demonstrated in recent operational periods. Following a discussion and review of NAVSPASUR operating parameters, a statistical analysis of system performance will be presented. This analysis will consist of data regressions performed by the GRAFSTAT and SAS programs imbedded in the Naval Postgraduate School mainframe computer. The final result of this effort will be to provide NAVSPASUR with an independently derived, statistically based means to predict future probabilities of success in detecting satellites of known radar cross section in operational orbits.

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NAVSPASUR Sensor Performance Study

by

Stephen F. Schaaf
Lieutenant, United States Navy
B.S., Virginia Military Institute, 1983

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September 1991

Author: Stephen F. Schaaf
Stephen F. Schaaf

Approved by: Donald R. Barr
Donald R. Barr, Thesis Advisor

Dan C. Boger
Dan C. Boger, Second Reader

R Panholzer
Rudolph Panholzer, Chairman,
Department of Space Systems

ABSTRACT

The Naval Space Surveillance Command (NAVSPASUR) has been in existence since the late 1950's. Operating with a formidable array of three large transmitters and six receiving stations, the command has carried out the mission of surveilling and cataloging all space objects in near earth orbit. To date, over 21,000 artificial satellites have been tracked and catalogued by the command. In order to document how effectively the fence has been accomplishing this mission, this thesis has been undertaken to provide NAVSPASUR with a statistics based measure of demonstrated system detection performance.

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My special thoughts throughout this effort go to my son Hunter, the finest son a man could ask for.

I. INTRODUCTION

The NAVSPASUR radar fence was built in 1958. It has since tracked over 21,000 space objects, over 6000 of which are in orbit today. The reason for existence and basic operating principles of the fence are best summarized in a paragraph taken from a study completed by Dr. S. H. Knowles of NAVSPASUR, Dahlgren, VA. [Ref. 5] In it he states:

Tracking and acquisition of artificial satellites that cooperate by transponding is a well-proven technique with many practitioners. However, for defense purposes it is of great value to be able to detect and determine an orbit for satellites with no cooperation or pre-information required. This important task is accomplished for our country, not by the large X-band parabolas usually associated with tracking, but by a radio fence of 217 MHz radiation located across the southern part of our country and operated by the U. S. Navy as the Naval Space Surveillance Command (NAVSPASUR). The NAVSPASUR system, unlike conventional tracking radars, uses sets of dipoles in an interferometer array to derive directly the angular position of each satellite that passes through the fence. Because of the laws of orbital mechanics, all satellites in 'parking' (i.e. thrustless) orbits that pass over CONUS must eventually pass through the NAVSPASUR fence and are detected with no requirement of pre-targeting or cooperation. This NAVSPASUR radar-interferometer system has remained essentially unaltered in concept for many years and has served as a mainstay of our country's satellite surveillance system.

Throughout its thirty plus years of operation, NAVSPASUR fence operators have lacked a formal, statistically proven measure of system effectiveness. Specifically, a study of demonstrated system capability to detect satellites of known radar cross section at certain operational altitudes is needed. It is the intention of the author to provide NAVSPASUR with information about detection performance and a model to predict system effectiveness in most operational regimes of interest.

The Chapter II herein deals with the background of the NAVSPASUR fence. An overall description of the fence is given and its associated radar properties are examined. Also, the logic tree under which the system collects data and its ramifications in terms of demonstrated system detection performance are discussed.

Chapter III presents the statistical analysis procedures applied to actual NAVSPASUR data in order to estimate probability of detection models. An overview of the data used as well as a discussion of dependent and independent variables as related to a final probability model are presented. It is hoped this will provide the statistically unindoctrinated reader with an appreciation for the method of estimating the probabilities of detection of future space platforms of interest.

Chapter IV is dedicated to exploratory data analysis performed in large part with the use of the GRAFSTAT program. The characteristics of the NAVSPASUR data set are examined closely in order to provide a basis for developing a more accurate model later in the thesis. While the exploratory data analyses do not provide exact probability of detection models, the graphical summaries presented show definite and informative trends in the data, which should be of interest to NAVSPASUR operators.

The Chapter V presents results of statistical analyses and modeling efforts performed with the SAS program. Individual parameters are estimated in a logistic regression model using our observed satellite data. The analyses were performed to relate detections and non-detections with independent variables, such as inclination, altitude, and RCS, in such a manner as to provide an accurate probability of detection model for many different detection regimes.

Chapter VI provides a summary of the results, as well as an indication of possible areas for continued study.

II. BACKGROUND

A. DESCRIPTION OF THE RADAR FENCE

The present NAVSPASUR transmitting system consists of three separate transmitters positioned on a great circle across the southern United States. The transmitting antenna at each site consists of a linear array of dipole elements aligned in a north-south direction. Each site transmits an unmodulated continuous-wave signal at a frequency (f) of 216.980 MHz, corresponding to a wavelength (λ) of 1.38 meters.

The 810 kW transmitter at Lake Kickapoo, Texas is the most powerful and the longest, consisting of eighteen separate collinear bays stretching 3200 m in the north-south direction. Each bay contains 144 elements spaced 1.27 m (0.92 λ) apart, except for bay #8 (numbering from north to south), which is split up by a road and consists of two half bays with 54 elements each. The end elements of adjacent bays are separated by 3.81 m. The distance between the elements at the road gap is 73.2 m. The Kickapoo transmitter is referred to as the Kickapoo complex, since it is created from two smaller nine-bay transmitters called North Kickapoo and South Kickapoo. Each half can be operated as an individual transmitter antenna.

The Gila River, Arizona and the Jordan Lake, Alabama transmitters each supply 45kW of power to single bay antenna arrays. The Gila River transmitter has 384 elements spaced 1.30 m (0.94 λ) apart, while the Jordan Lake transmitter has 256 elements spaced 1.22 m (0.88 λ) apart.[Ref. 3]

A series of six large antenna arrays comprise the receiver segment of the fence. These units, their locations, and specific characteristics are shown in Table 1 [Ref. 8].

TABLE 1. NAVSPASUR RECEIVING STATION ARRAY DIMENSIONS

Name/Location	No. Arrays/Length (ft)	Alert Antenna(ft)
San Diego, CA	12 / 400	1600
Elephant Butte, NM	22 / 1200	Dual 4800
Red River, AR	12 / 400	1600
Silver Lake, MS	12/ 400	1600
Hawkinsville, GA	22/ 1200	Dual 4800
Fort Stewart, GA	12/ 400	1600

This group of transmitters and receivers comprise the hardware elements of the NAVSPASUR fence. By combining the gains inherent in very large antennas with the coverage of wide geographic regions, the fence is able to provide extraordinary surveillance of space objects crossing continental United States territory. The relative spacing of these antennas is shown in Figure 1 [Ref. 2].

B. RADAR THEORY

The NAVSPASUR fence can be most easily visualized as a fan of electromagnetic energy similar in geometry to that depicted in Figure 2. In reality, this system merely represents the confluence of the individual transmitting and receiving stations. It is, therefore, beneficial to examine a single station. As an example, consider Kickapoo to be the transmitter and San Diego to be the receiver.

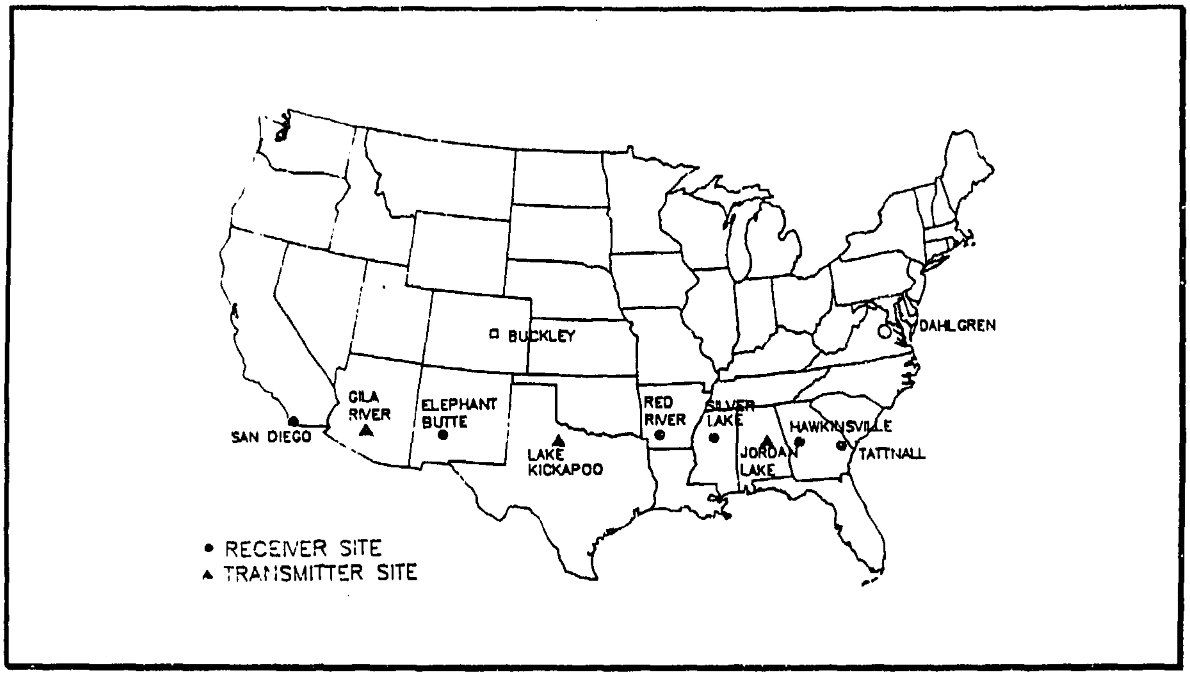

Figure 1. Locations of NAVSPASUR Transmitter and Receiver Sites

A recent study of fence radiation properties completed by Mr. Guy H. Chaney examined the theoretical limitations of the Kickapoo transmitting station [Ref. 3]. While many assumptions were made for ease of calculation, the basic constants and link calculations used were all consistent with accepted radio theory. The largest area of uncertainty in the calculations appears to be in the determination of a consistent gain value for the antennas. With this limitation in mind, a modified version of Chaney's work will be presented in order to demonstrate system theoretical range.

The applicable specifications of the NAVSPASUR fence are as follows:

let, k = Boltzman's constant = 1.38E-23

F_n = Receiver noise figure = 1.58 dbW(assumed constant for fence)

T_r = Receiver Noise Temperature = T_o(Fn-1) = 290K(1.58-1)

T_{sky} = Sky noise temperature = (assumed 100K)

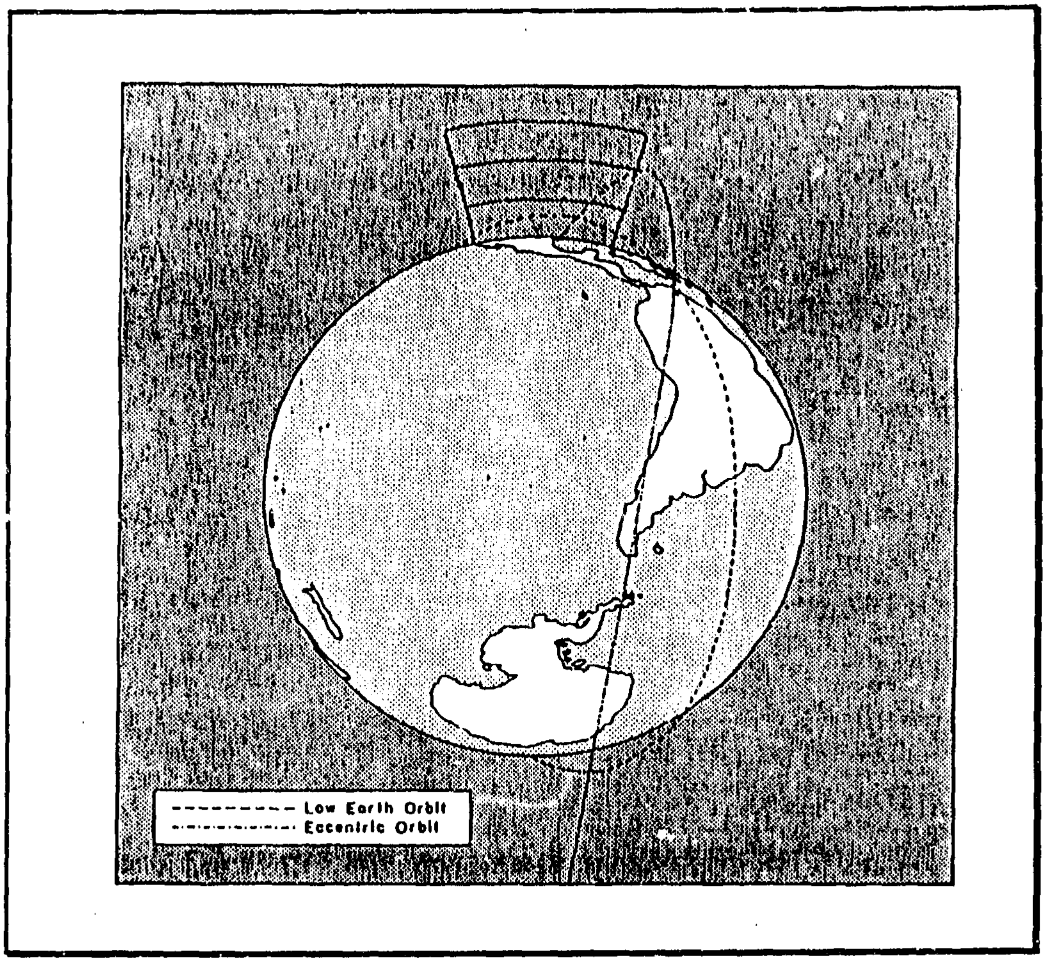

Figure 2. Depiction of Fence Energy Field as Seen from Space

B = Receiver Bandwidth = 36.6 Hz (low altitude reception)

P_t = Transmitter power = 766.8 kW (number varies slightly per reference)

N = receiver noise power = $k(T_r + T_{sky})B$ = -188.7 dbW

N_t= Number of dipoles in transmitting antenna = 2556

$$G_t = \text{Transmitter gain} = 3 + \frac{10\log[2^*N_t]^*3}{\log[2]} = 40\text{db} = 10000$$

N_r = Number of dipoles in receiving antenna = 96

$$G_r = \text{Receiver gain} = 3 + \frac{10\log[2^*N_r]^*3}{\log[2]} = 25\text{ db} = 316$$

w = Fence operating wavelength = 1.38 m = 4.528 ft

RCS = Radar cross section of target satellite in square meters

P_r = Power required at the receiver for 7 dB signal to noise ratio = N +

$$S/N = -188.7 \text{ dbW} + 7\text{dB} = -181.7 \text{ dbW} = 6.76E\text{-}19$$

losses = 2 dB (due to antenna, coupling and atmosphere) = 1.58

Conversion factor (meters to nautical miles) = .0005468

R = Maximum theoretical range (nm)

For the maximum theoretical range equation, Chaney used the form:

$$R^4 = \frac{(P_t)(G_t)(G_r)(w^2)(RCS)}{[(4)(\pi)]^3(P_r)(\text{losses})}(.0005468)$$

The resultant equation obtained when the values shown above are entered is given by:

$$R = (RCS)^{.25}(3687).$$

A plot of maximum range for a range of values of RCS is shown in Figure 3.

C. DATA COLLECTION

Some reduction in the volume of observational data is inherent in the computer processing of NAVSPASUR data. The logic flow chart in Figure 4 illustrates the requirements for a successful observation to be recorded.

As can be seen in Figure 4, there are several data "pigeon holes" into which some received data may fall and thus be excluded from the useable database. These are due to imbedded software constraints designed to weed out noisy or extraneous signals. One example of data rejection is that of a satellite being illuminated by a transmitter at less than 6 degrees above the horizon. Due to ground clutter and receiver interference problems, this region has been excluded from the acceptable observation category. Bad resolution of a crossing target will also suspend an observation. This phenomenon, which may occur for a number of reasons, including poor doppler resolution, timing or cosine errors, results when the computer fails to match a definite radar signature. A final data sink is due to buffer overflow. The buffer holds information that awaits CPU processing, and occasional information loss occurs when storage capacity is exceeded. [Ref. 11]

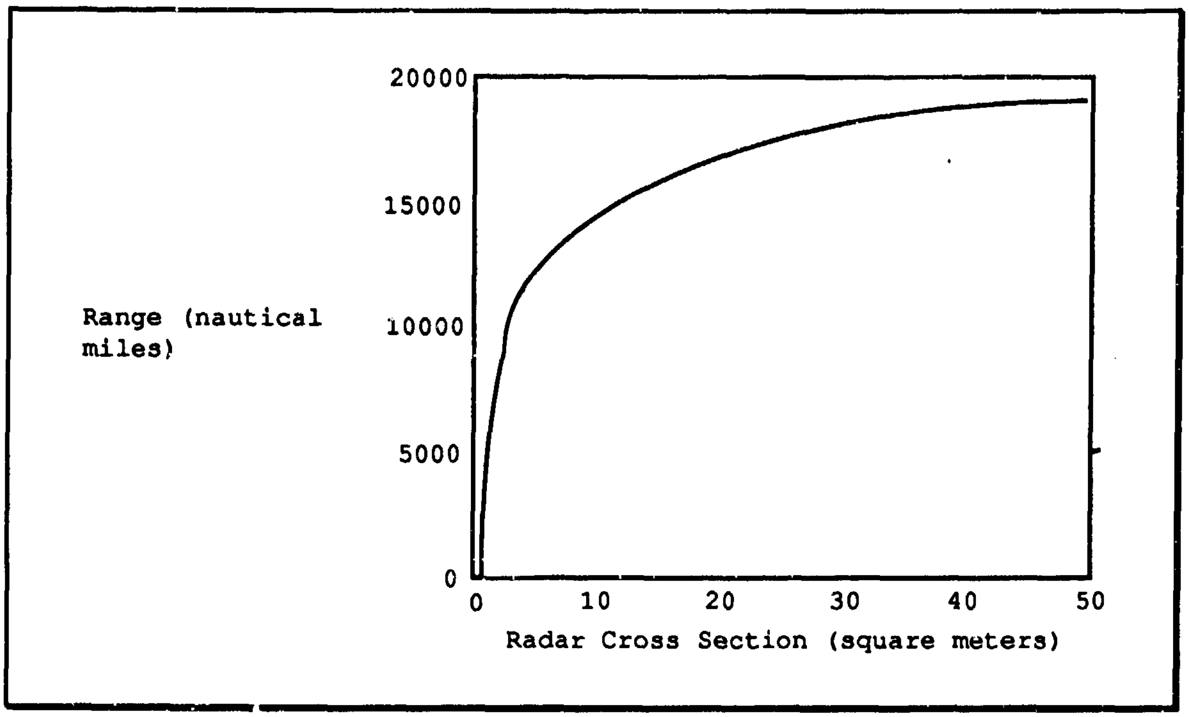

Figure 3. Maximum Theoretical Ranges as a Function of Radar Cross Section

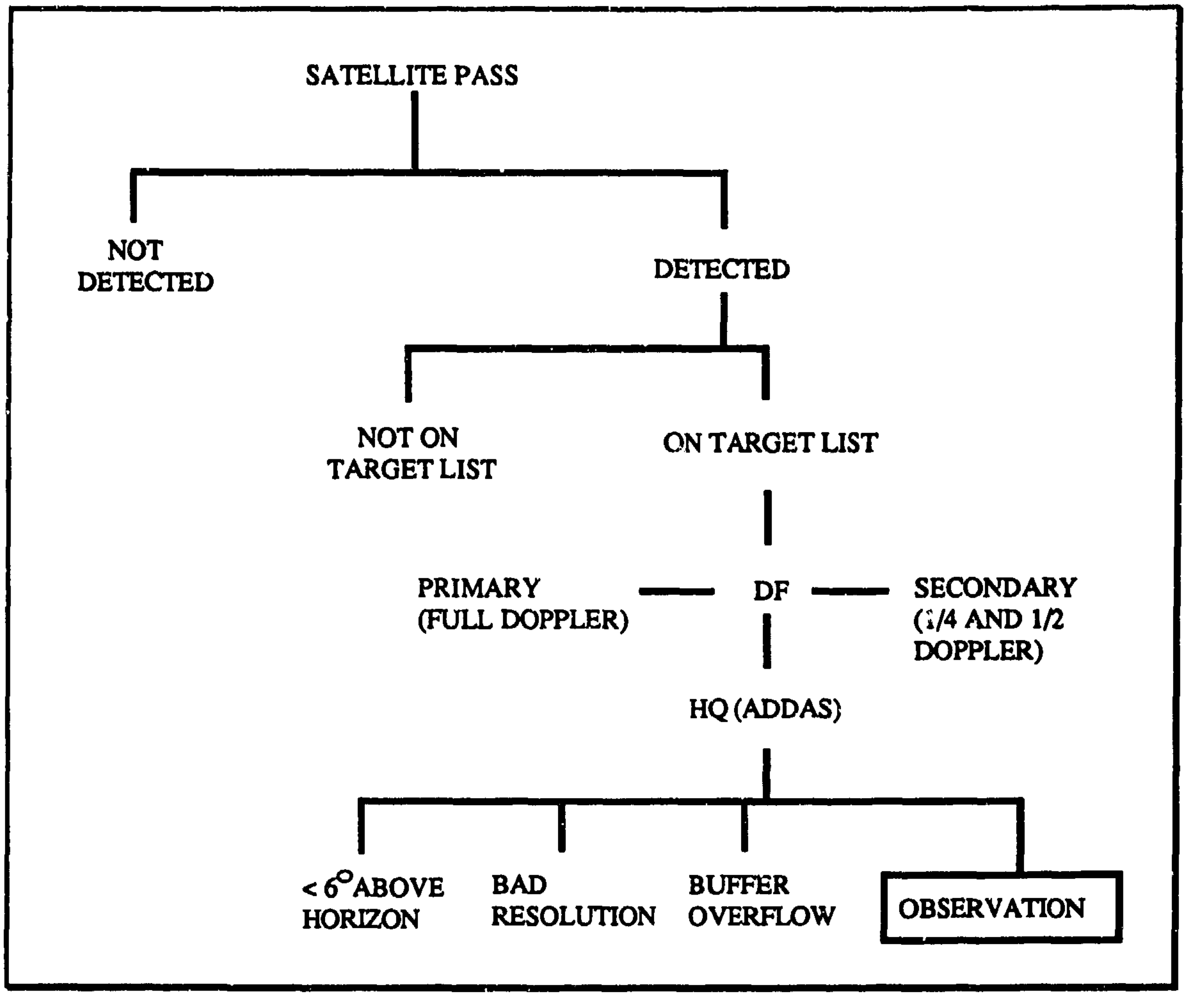

Figure 4. Logic Flow for NAVSPASUR Received Data

These limitations to the current NAVSPASUR system are certainly relevant and contribute in a nontrivial fashion to the loss of received data. They are discussed here only to provide background as to why the fence system records a less than perfect detection rate. The accurate estimation of the level of fence detection performance is the goal of this thesis effort.

III. THE MODEL

A. OVERVIEW

The NAVSPASUR fence and its associated data collection elements constitute a system that lends itself to statistical analysis. The voluminous quantity of orbit prediction and observation data, exceeding 37,000 events per day, provide a significant base from which system performance may be analyzed in terms of relevant parameters. For example, in light of the total number of observations available, it is possible to examine system performance with regards to specific satellite radar cross section values or categories. As an example, system performance against a group of small satellites (approximately 1 square meter RCS) is analyzed below. Other primary variables of concern in the estimation of system detection performance are altitude and longitude at time of detection, and inclination of the satellite orbits. In our analyses of these data, two statistical and graphics packages were employed.

The SAS (Statistical Analysis System) package was used to examine the effects of several parameters on system performance. The SAS procedure LOGIST was used to perform logistic regression. The object of logistic regression is to fit models for Bernoulli (detect/no detect) data. A review of logistic regression follows in the next section. The primary output of the SAS procedure LOGIST is a model giving predicted probability of satellite detection for any combination of crossing satellite parameter values. The GRAFSTAT package was also used in this research in order to provide comparisons with

SAS calculations, and to provide visually descriptive three dimensional graphics. Professor Peter Lewis of the NPS Operations Research Department wrote specific routines allowing GRAFSTAT to perform weighted regression with logit transformed relative frequencies. This gives statistical results similar to those from the SAS program, but for restricted parameter sets. GRAFSTAT provides superb graphics functions. These graphics provide concise summaries of system performance. In what follows we review the statistical basis of the analysis we performed.

B. LOGISTIC REGRESSION

A magnetic computer tape received from Mr. Robin Smith of NAVSPASUR contained all satellite prediction and observation data for 24 April 1991. Each of the 37,181 records in the tape consist of thirteen specific fields. The first seven of these fields include time, catalogue number, radar cross section, eccentricity, inclination, altitude, and longitude of a satellite when crossing the fence. The final six columns are the indications of successful or unsuccessful detection for each NAVSPASUR receiving station. If a station indicates a zero for a given satellite pass, the satellite was out of the station's line of sight and should not count as a receiver success or failure. (This indication will obviously be excluded from the analysis.) On the other hand, a one indicated by the station means a satellite should have been in view of the station, but was not detected. This should be regarded as a receiver failure. Finally, if a given station indicates a two or greater (there is some variability in this value due to time spent within the energy field), the station has successfully detected the predicted satellite pass. Given this Bernoulli (success or no success) type of response data, logistic regression is a good candidate procedure for arriving at a probability of success (detection) model.

Logistic regression, like most model building techniques used in statistics, has the simple goal of finding the best fitting and most parsimonious, yet physically reasonable model to describe the relationship between an outcome (dependent variable) and a set of explanatory (independent) variables. The independent variables are frequently called covariates. What differentiates logistic regression from normal linear regression modeling is that the outcome variable in logistic regression is binary (success/failure). This difference between logistic and linear regression is reflected both in the choice of a parametric model and in the assumptions. Aside from this difference, the methods employed in an analysis using logistic regression follow the same general principles used in linear regression. [Ref. 5]

For the sake of clarity, linear regression may be briefly described as follows. Suppose there is a relationship between two variables, such that a linear association is suspected. More variables may exist, as is the case with this study. However, for simplicity, we will discuss only the single variable case. This relationship could be described by the equation:

$$a + bx = y + \text{error}$$

In an environment where y is a continuous dependent variable, many experimental results could be compiled to estimate a and b, providing the best fit for the given relationship. This technique of curve fitting, known as linear regression, can be extended to multivariate and nonlinear relationships.[Ref. 5] It is not appropriate, however, for fitting models where the dependent variable is binary, as is the case with the NAVSPASUR detect/no detect response variable.

As mentioned previously, two major differences exist between linear regression and logistic regression. The first difference concerns the nature of the relationship between the dependent and independent variables. In a regression problem the key quantity is the mean (expected) value of the dependent variable, given the values of the independent variables. This quantity is known as the conditional mean and can be expressed "E(Y/x)" where Y is the dependent variable and x denotes the value of the independent variables. E(Y/x) can be stated in English as "the expected value of Y, given the value of x." In the case of linear regression, where E(Y/x) can take on any real value as x ranges over some Euclidean space, we can describe this relationship simply by the equation [Ref. 5]:

$$E(Y/x) = \beta_0 + \beta_1 x$$

For predicting probabilities, however, E(Y/x) should always be equal to or greater than 0 *and* less than or equal to 1. In the NAVSPASUR data set, where the outcome variable is dichotomous, a plot of E(Y/x) would be S-shaped when plotted against x, resembling a cumulative distribution of the independent variables. In other words, as the conditional mean approaches 0 or 1, the change in E(Y/x) per-unit change in independent variables becomes progressively smaller. The precise shape of this S-curve is determined by the relationship of independent variables to the outcome variable. Given this particular modeling environment, it would possible to select from several well known functions in order to attain an acceptable linearizing transformation of E(Y/x). The logit transformation is chosen because: (1) it is an extremely flexible and easily used function from a mathematical point of view, and (2) it lends itself to physically meaningful interpretation.

As stated previously, the first difference between logistic regression and linear regression is that the dependent and independent variables are related differently. This difference is reflected in the basic form of the logistic regression equation:

$$\pi(x) = E(Y/x) = \frac{\exp(\beta 0+\beta 1x)}{1 + \exp(\beta 0+\beta 1x)} ,$$

where $\pi(x)$ can be interpreted as the probability that y equals 1, given x.

Using the logit transformation on $\pi(x)$, defined by

$$g(x) = \ln\left[\frac{\pi(x)}{1-\pi(x)}\right]$$

it easily follows that

$$g(x) = \beta_0 + \beta_1 x .$$

As can be seen, the logit transformed probabilities of detection, $\pi(x)$, may have the desirable properties of the linear regression model, and may be continuous, depending on the domain of g. Logistic regression uses the mathematical properties of the above exponential relationship to provide an estimator for the expected values, E(Y/x). Reference 5 contains a thorough explanation of the statistical assumptions in logistic regression. It states:

The second difference between the linear regression and logistic regression models concerns the conditional distribution of the dependent variable. In the linear regression model it is assumed that an observation of the dependent variable may be expressed as:

$$y = E(Y/x) + \varepsilon$$

The quantity e is called the error and expresses an observation's deviation from the conditional mean. The most common assumption is that e follows a normal distribution with mean zero and some variance that is consistent across levels of the independent variable. It follows that the conditional distribution of the dependent variable given x will be normal with mean $E(Y/x)$, and a variance that is constant. This is not the case with a dichotomous outcome variable. In this situation we may express the value of the outcome variable given x as $y = \pi(x) + e$. Here the quantity e may assume one of two possible values. If $y = 1$ then $\varepsilon = 1 - \pi(x)$ with probability $\pi(x)$, and if $y = 0$ then $\varepsilon = -\pi(x)$ with probability $1 - \pi(x)$. Thus, ε has a distribution with mean zero and variance equal to $\pi(x)[1 - \pi(x)]$. That is, the conditional distribution of the outcome variable follows a binomial distribution with probability given by the conditional mean, $\pi(x)$.

A summary of properties of the logistic regression model, which we believe are appropriate for the NAVSPASUR performance study is:

- The conditional mean of the observed response variable must be formulated to be bounded between zero and one;
- The binomial distribution describes the distribution of the errors and is the statistical distribution upon which the analysis is based; and
- The principles that guide an analysis using linear regression apply in logistic regression.

C. SIGNIFICANCE OF THE MODEL

The SAS package, like many other well known software packages, provides the user an assessment of the quality of the proposed model. A primary aspect of this testing is the determination of whether the independent variables in the model are "significantly" related to the outcome variable. In other words, does the model that includes the variable(s) in question tell us more about the outcome variable than does a model that does not include that (those) variable(s)? Reference 5 provides an answer to this question, stating:

> The observed values of the response variable predicted by each of two different models are compared; the first with and the second without the variable in question. The mathematical function used to compare the observed and predicted values depends on the particular problem. If the predicted values with the variable in the model are better, or more accurate in some sense, than when the variable is not in the model, then we feel that the variable in question is "significant".

The SAS program tests for such significance through the following procedure. The log likelihood equation for the observed outcome with parameters in the vector β in the model is generated [Ref. 5]:

$$L(\beta) = \Sigma\{y_i \ln[\pi(x_i)] + (1 - y_i)\ln[1 - \pi(x_i)]\}$$

As can be seen, for independent variable combinations where $y_i = 1$ the contribution to the log likelihood function is $\log[\pi(x_i)]$, and where $y_i = 0$ the contribution is $\ln[1 - \pi(x_i)]$. SAS uses an iterative numerical algorithm to find a value of β for each independent variable that maximizes $L(\beta)$ for the data set where i ranges from 1 to total sample size, 37,181 in our case. The given variables are examined using the equation:

$$G = -2\ln\left[\frac{\text{(likelihood without the variable)}}{\text{(likelihood with the variable)}}\right]$$

The p-value corresponding to this test statistic, G, is determined with a chi-square (χ^2) distribution with a certain number of degrees of freedom . If this p-value is relatively small, we have convincing evidence that the variable in question is significant.[Ref. 7] That is, the null hypothesis that the corresponding coefficient is zero should be rejected. This testing was carried out for each candidate independent variable in our model.

IV. EXPLORATORY ANALYSIS

A. INITIAL DATA EXPLORATION

Before undertaking full scale logistic regression analysis with the NAVSPASUR database we examined some general trends of the data. Knowing the ranges of parameters under which the majority of predictions and observations fall is a useful input to more concise modelling. The Grafstat package was used to provide graphical pictures of data patterns. Two dimensional data density diagrams were quickly processed and all displayed in Figures 5, 6, and 7. The concept of the data density diagram is simply that the area under the curve will integrate to a value of 1, and most of the activity of interest occurs in regions where the curve is highest. In keeping with this principle, Figure 5 shows that the vast majority of satellite activity within 3000 nautical miles of earth. Figures 6 and 7 are similar; they depict NAVSPASUR data density as functions of different variables. Figure 6 shows that the aperature for the vast majority of tracking data falls between 55 degrees West longitude and 143 degrees West longitude. Figure 7 shows that most satellites range between .003 m^2 RCS and 8.7 m^2 RCS. Fitted multivariate detection prediction models will be most accurate over these high density domains of satellite activity.

Statistical characteristics of variables in these data density diagrams can be found in Appendix A. This information can be used to determine the parameter ranges under which fence detection performance was most frequently tested.

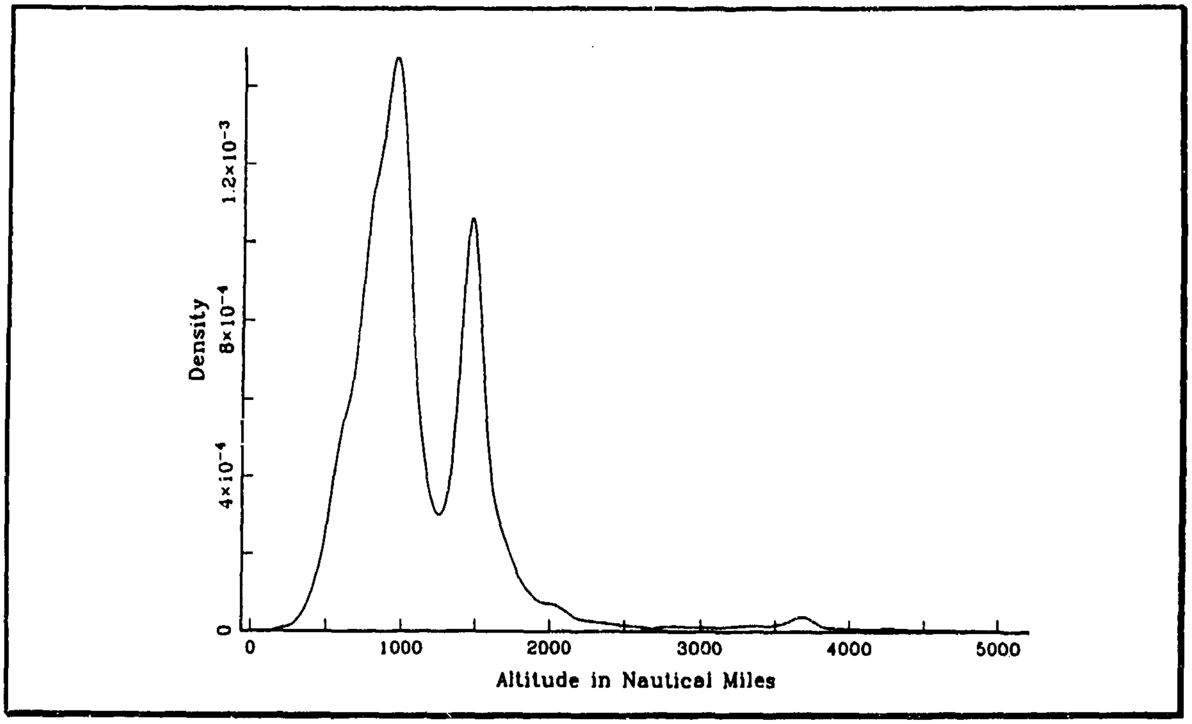

Figure 5. Data Density as a Function of Altitude

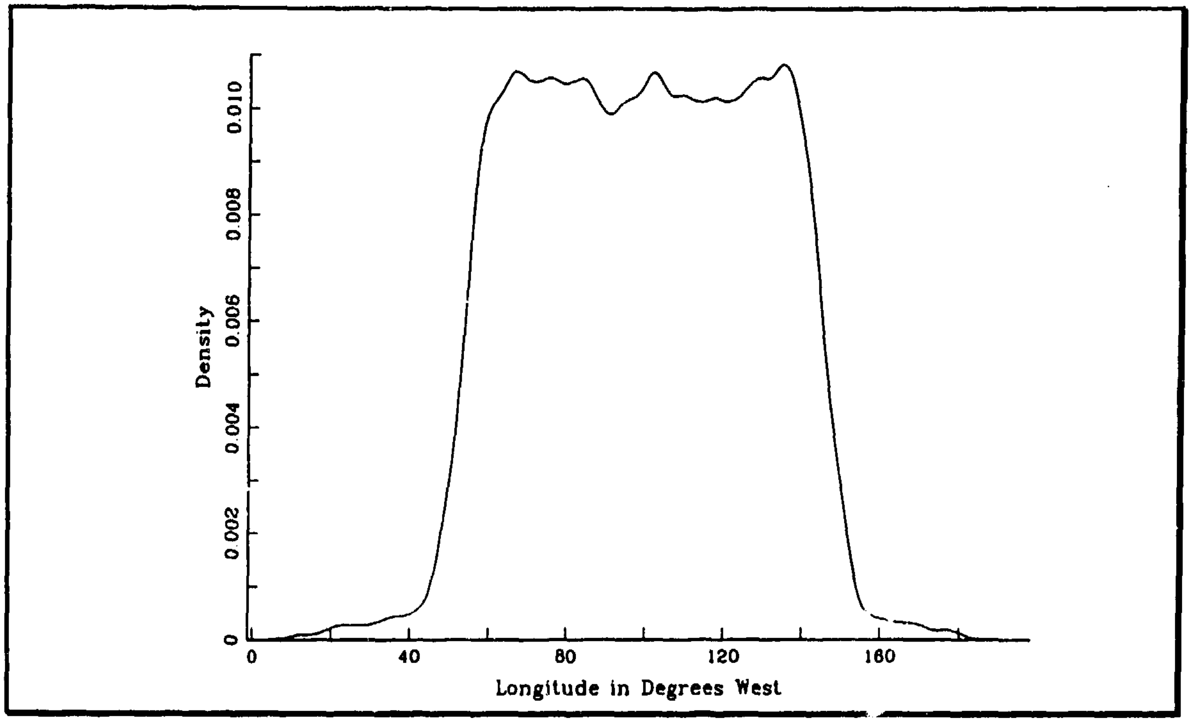

Figure 6. Data Density as a Function of Longitude

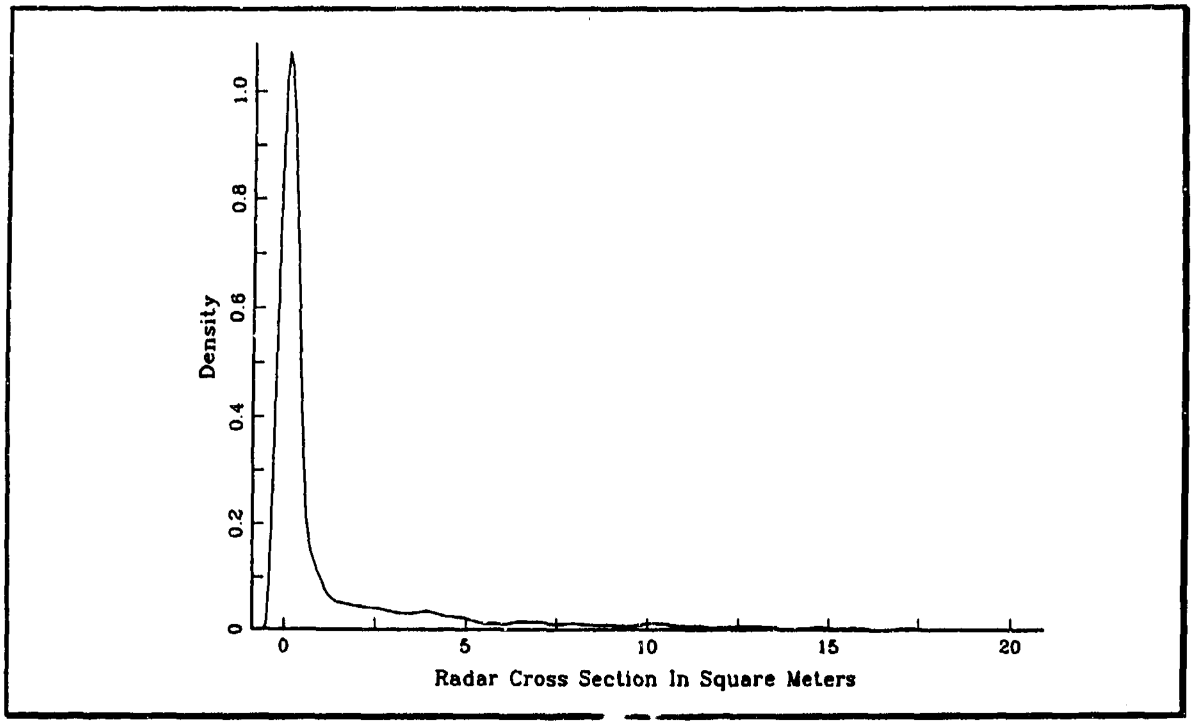

Figure 7. Data Density as a Function of Radar Cross Section

B. DETECTION ANALYSIS

During the preliminary GRAFSTAT analysis, emphasis was placed on the effects of longitude and altitude on probability of satellite detection. We were then able to create a three dimensional graphics depiction of $\pi(x)$, the probability of satellite detection, as a function of these two variables. We were able to approximate calculations of the logistic regression relationship of detections and nondetections using GRAFSTAT. The resultant probability of detection graphics will be shown to correlate closely with the formal logistic regression results.

1. Satellites of 1 M^2 RCS: Entire System

In a first run of the Grafstat package, satellites of radar cross sections ranging from .5 m^2 to 1.5m^2 were considered. The relative frequency with which the fence provides at least one receiver detection for each predicted fence penetration of this satellite group was calculated. The specific APL coding required for this run is shown in Appendix B. Figure 8 is a surface plot of the probability of detection function estimated for this group, ignoring inclination. Figure 9 is the contour plot for the same function. In this example it can easily be seen that sensor performance is best near the geographic center of the fence and begins to fall off with increasing altitude and off center longitude penetration.

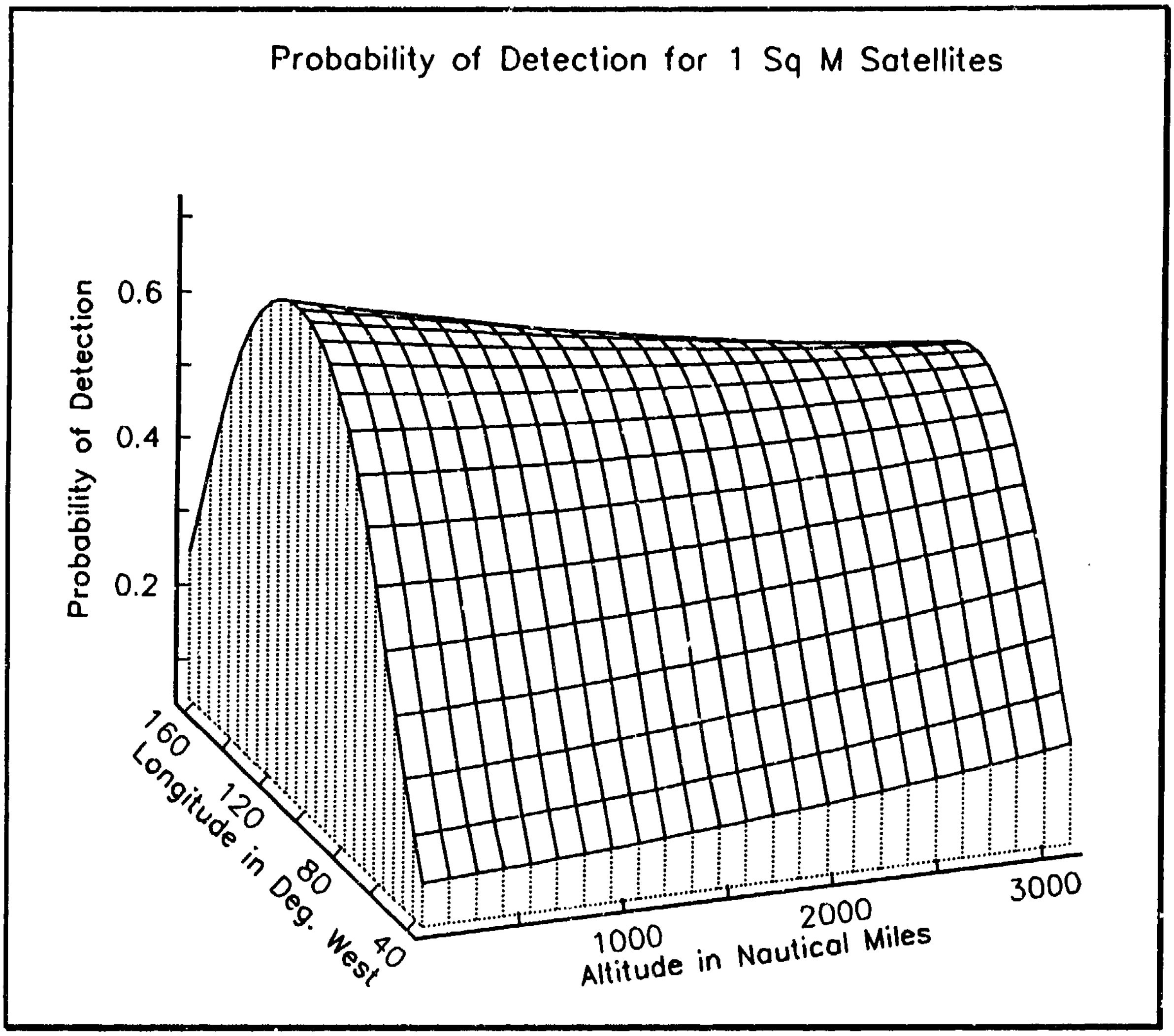

Figure 8. Surface Plot of Detection Performance

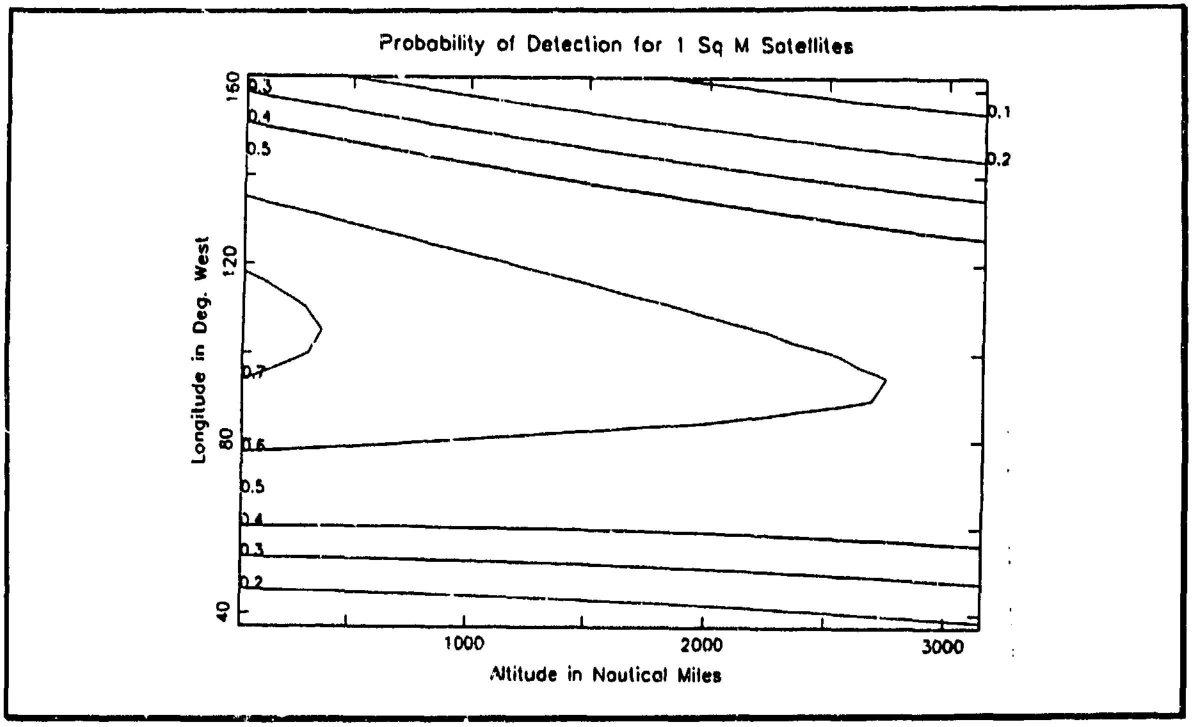

Figure 9. Contour Plot of Detection Performance

2. Satellites of 1 M^2 RCS: Individual Stations

In order to analyze the fence performance and the performance contribution of each of the individual receiver stations, analyses was completed for each station except for Elephant Butte, which was off line for maintainence throughout 24 April, 1991. Beginning with Figure 10, the surface and contour plots of estimated approximate detection performance for San Diego, Red River, Silver Lake, Hawkinsville, and Tattnall are presented. These plots show several aspects of the NAVSPASUR data set. Firstly, it can be noted that detection performance for each station is at a maximum directly over the respective station. For example, San Diego's performance appears best at 117 degrees west longitude, almost directly overhead the receiver. Also note that the estimated probability of detection surface plots for the individual stations show performance inferior to that of the fence taken as a whole, an indication of the benefit of the complementary nature of fence performance. Finally, the fact that independent analyses of data for five different stations gave similar and intuitively expected results suggests the statistical methods and the data set used in this preliminary analysis were consistent.

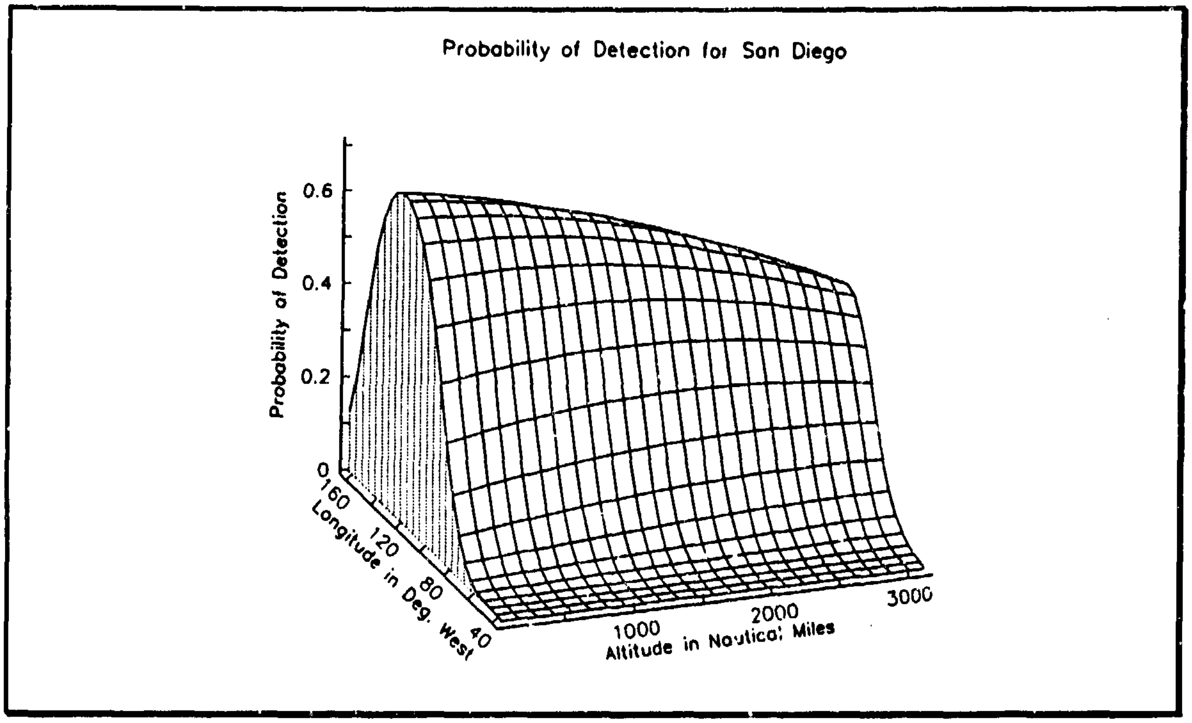

Figure 10. Surface Plot for San Diego

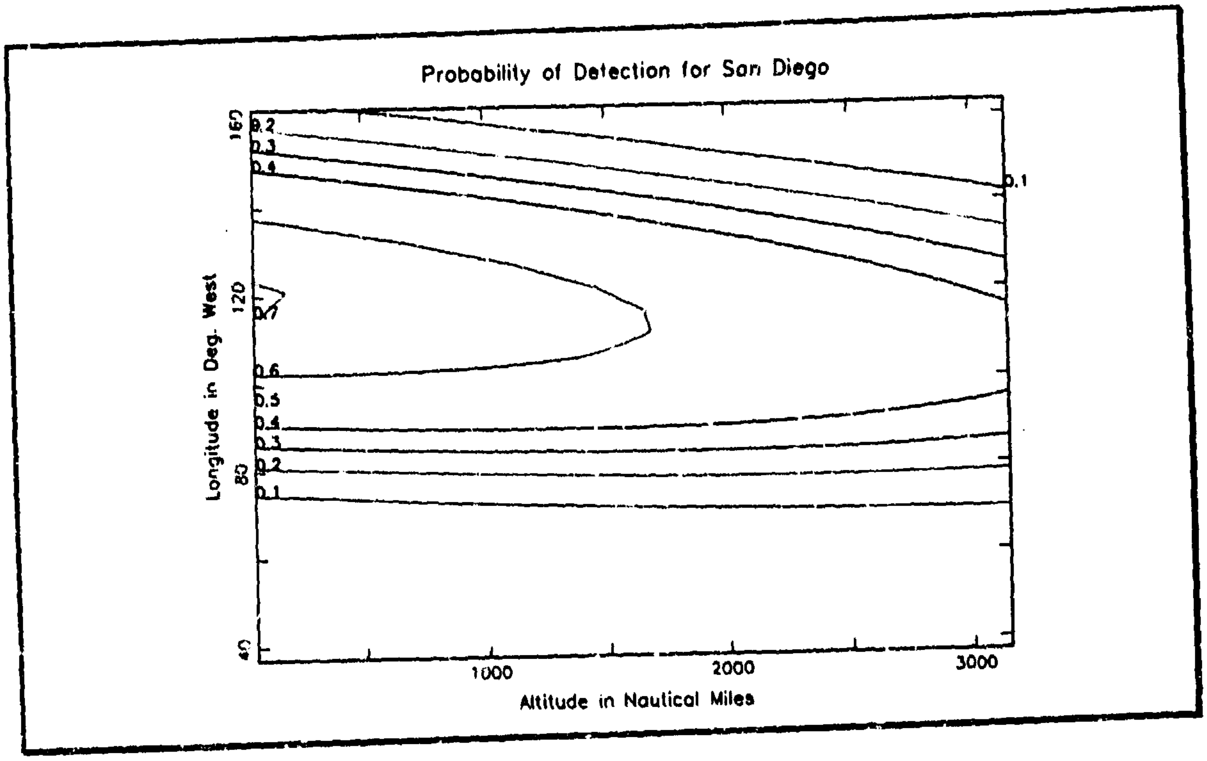

Figure 11. Contour Plot for San Diego

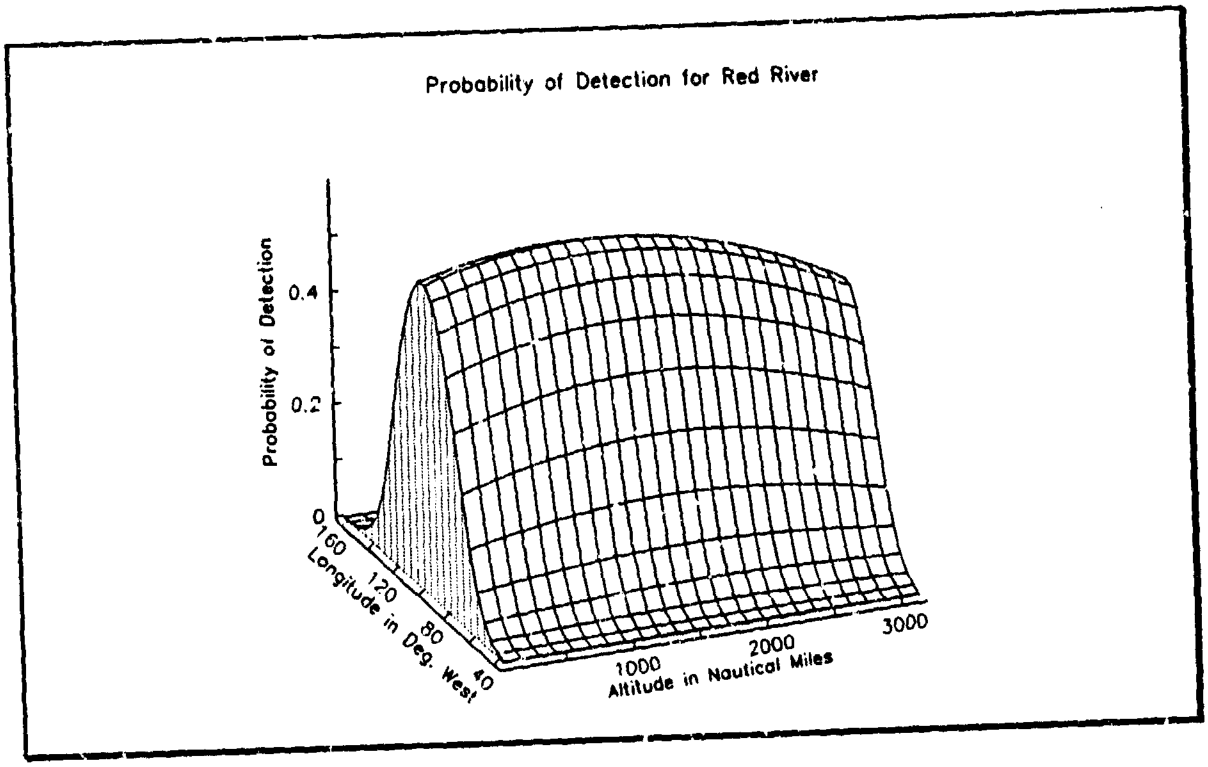

Figure 12. Surface Plot for Red River

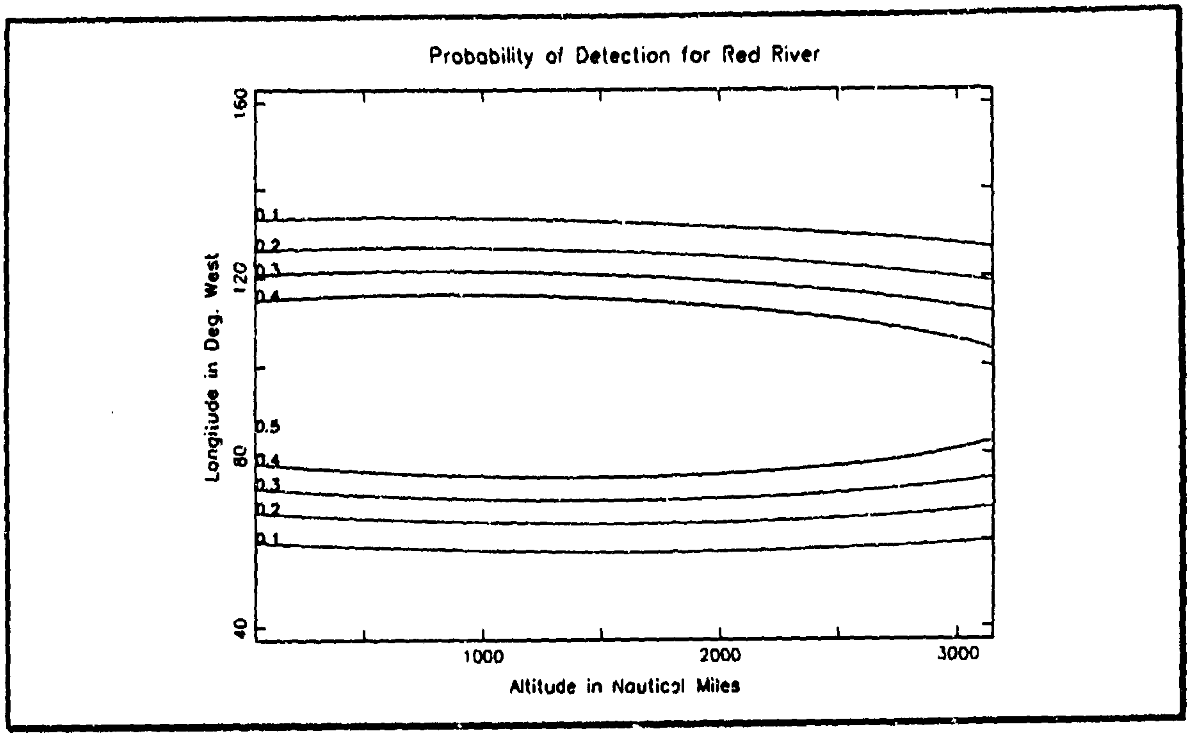

Figure 13. Contour Plot for Red River

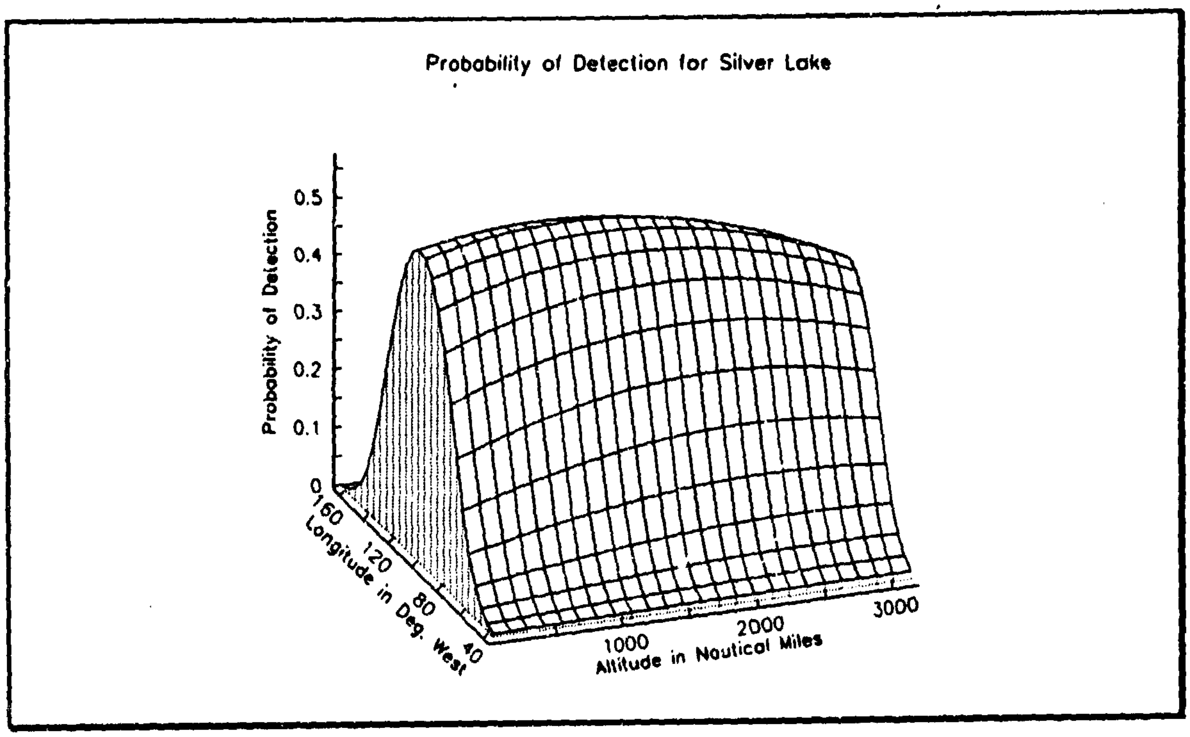

Figure 14. Surface Plot for Silver Lake

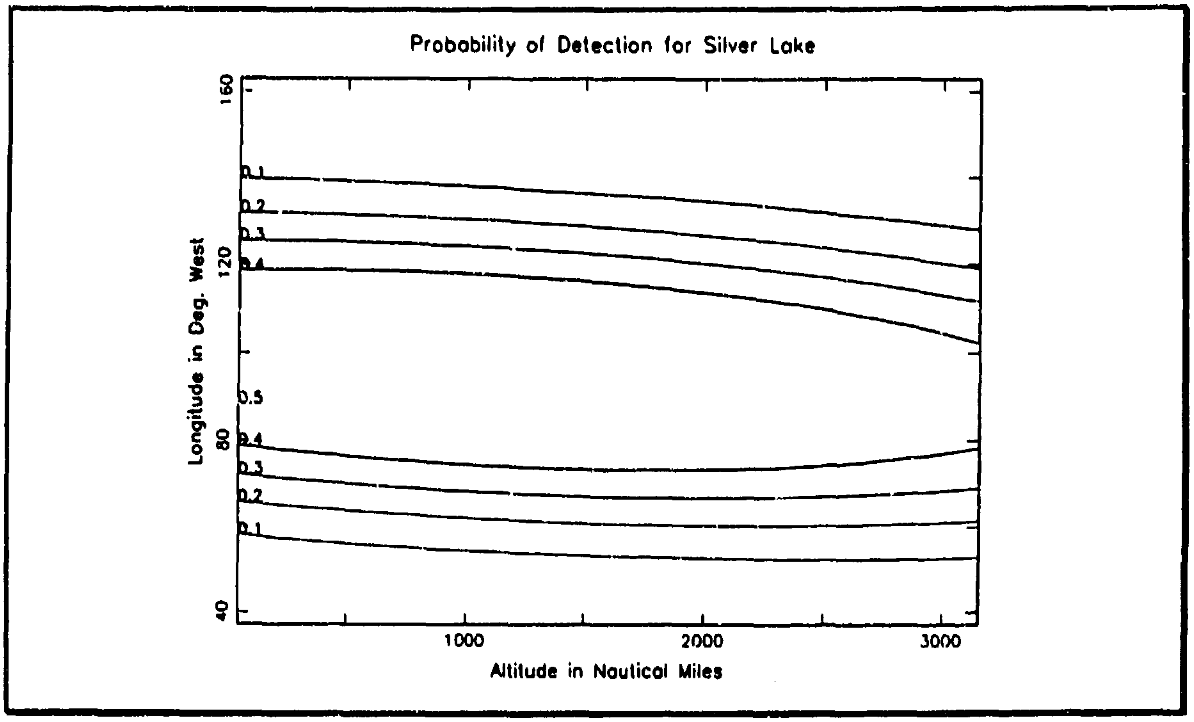

Figure 15. Contour Plot for Silver Lake

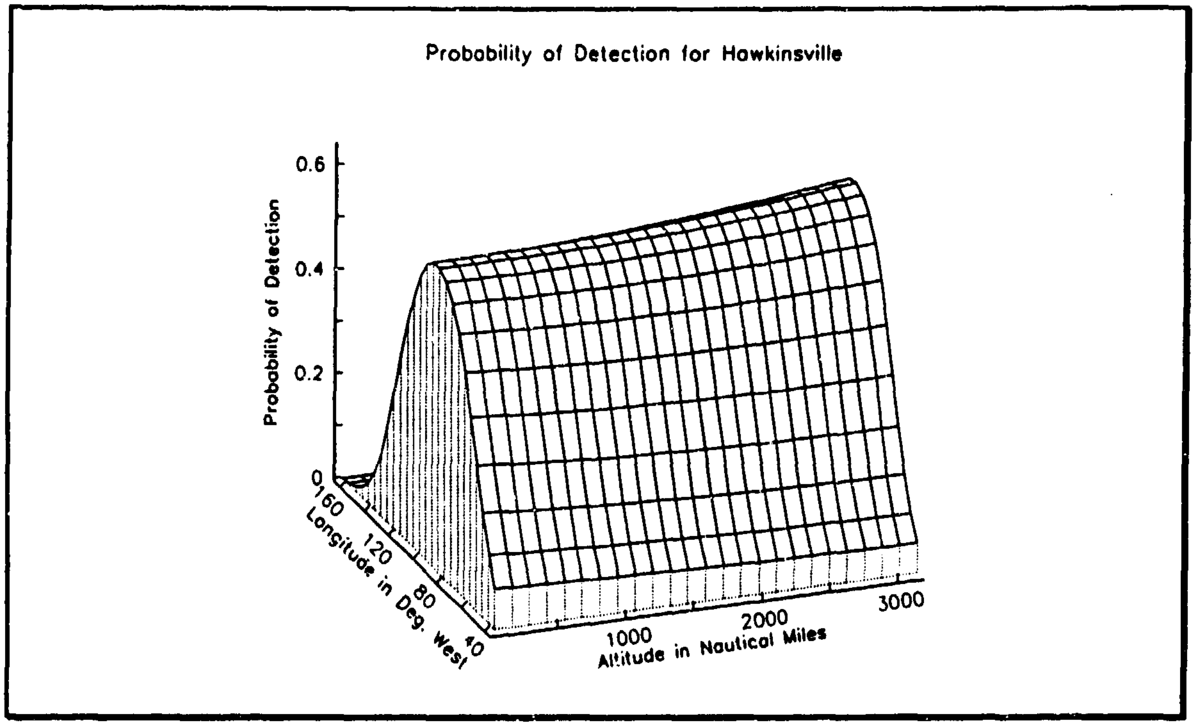

Figure 16. Surface Plot for Hawkinsville

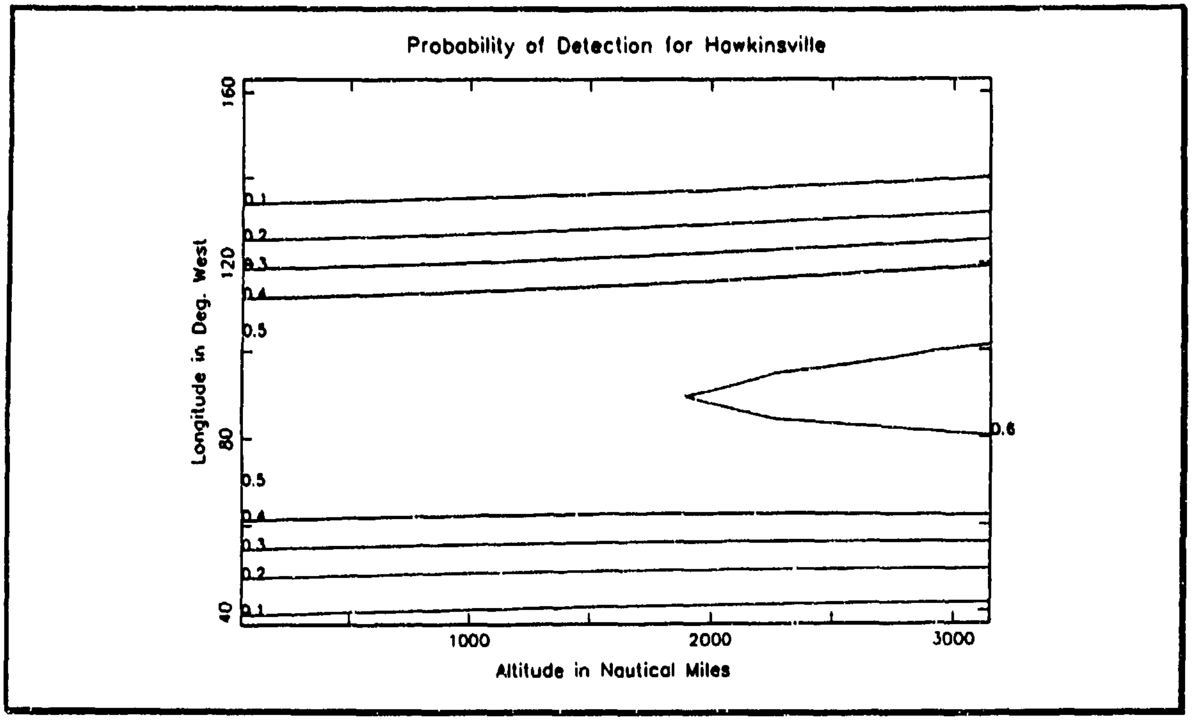

Figure 17. Contour Plot for Hawkinsville

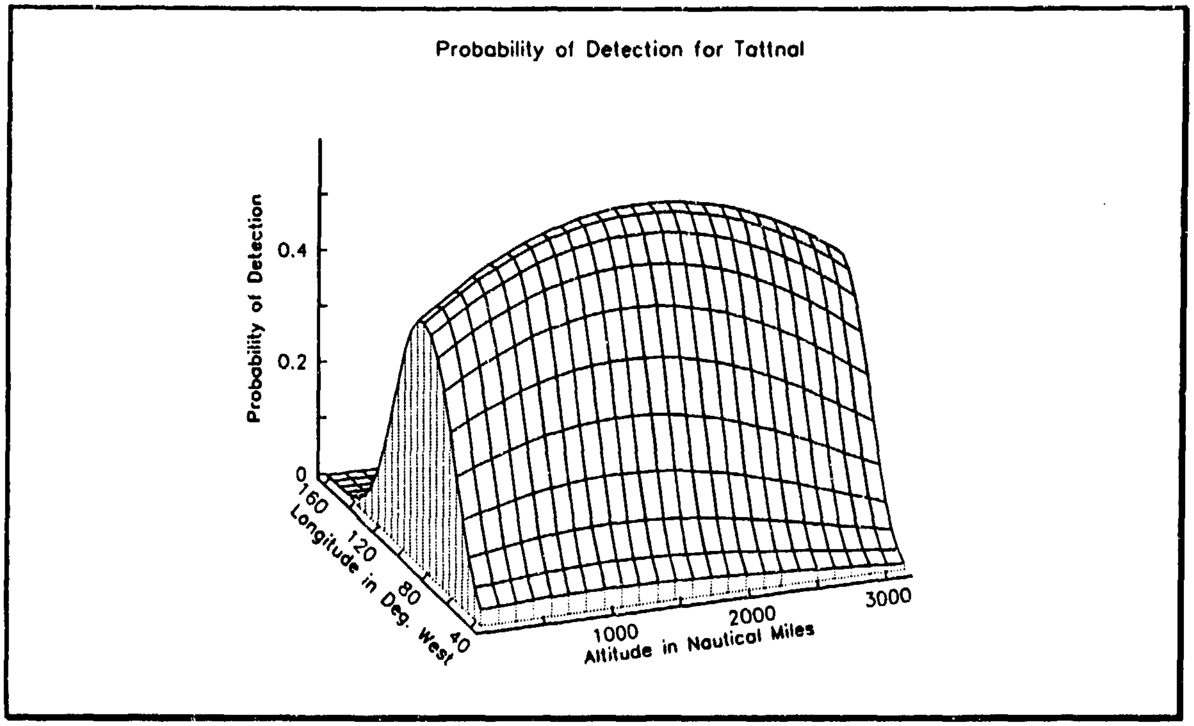

Figure 18. Surface Plot for Tattnal

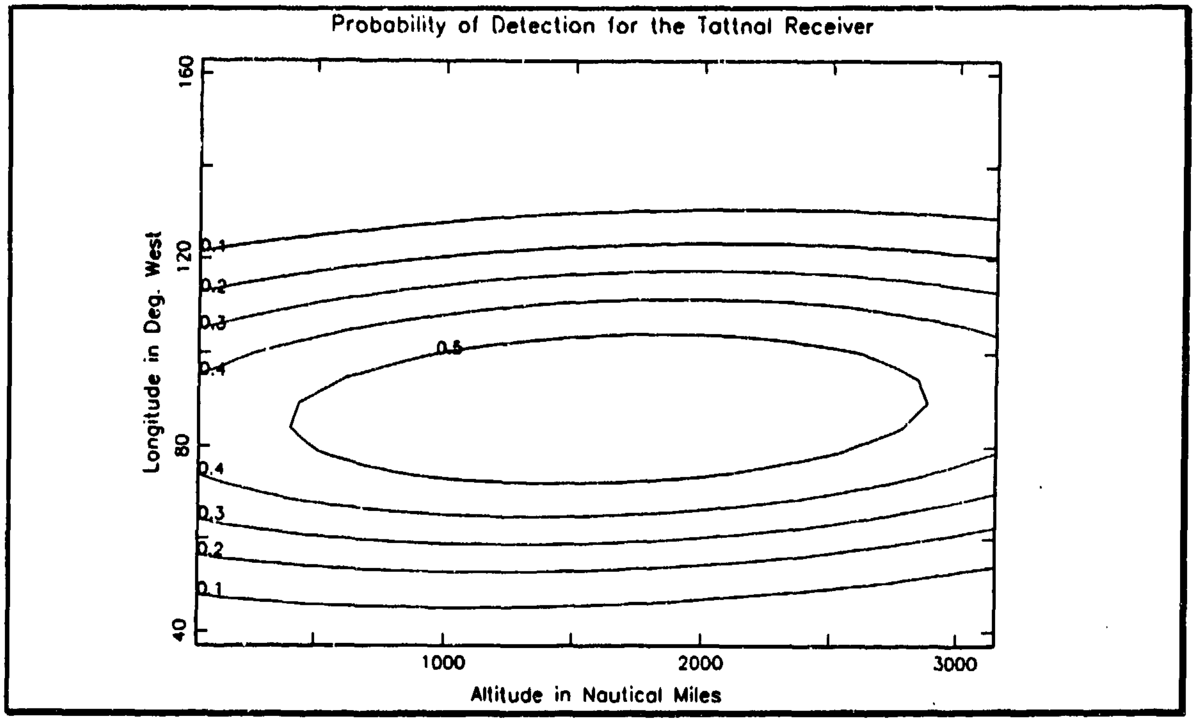

Figure 19. Contour Plot for Tattnal

V. LOGISTIC REGRESSION ANALYSIS

A. MODELING CONSIDERATIONS

Initial probability of detection modelling was completed in a largely automated fashion through the use of the SAS statistical package. Input setup included data editing and setting proper input variable relationships. Data editing included the setting of gates deemed appropriate in order to create a model more tuned to the large majority of NAVSPASUR data. Specifically, a gate was applied to take into account only the central 95% of the recorded data as discussed in the exploratory analysis portion of the analysis. While this eliminates some interesting extreme values in the data set, it should provide smoother and more accurate modelling of fence performance for regimes of greatest interest. The second important aspect of properly setting up the SAS package involves development of reasonable independent variables, based on scientific attributes and relationships. References 3 and 4 discuss the radiative properties of fence energy, suggesting that polynomial values might best model the input variables. The variables included in our model are assumed to have the following relationship:

$$g(x) = \beta_0 + \beta_1(ALT) + \beta_2(ALT)^2 + \beta_3(RCS) + \beta_4(LONG) + \beta_5(LONG)^2 + \beta_6(ARCCOSINC),$$

where,

ALT = Satellite altitude in nautical miles

RCS = Satellite radar cross section in square meters

LONG = Satellite longitude in degrees west

ARCCOSINC = Arcosine of satellite orbital inclination in radians

The maximum likelihood estimate of $\hat{\beta}$ determined through logistic regression, given this assumed relationship, was:

Parameter Estimate	Value	p-Value
$\hat{\beta}_0$	-19.99357495	0.0000
$\hat{\beta}_1$	0.00100492	0.0000
$\hat{\beta}_2$	-0.00000024	0.0001
$\hat{\beta}_3$	0.39842267	0.0000
$\hat{\beta}_4$	0.38942569	0.0000
$\hat{\beta}_5$	-0.00196066	0.0000
$\hat{\beta}_6$	-0.17300374	0.0148

The following section details how the $\hat{\beta}$ coeffecients are used to derive a probability of detection value for a hypothetical target satellite. Additionally, it is of value to note that the p-values associated with each of the $\hat{\beta}$ coeffecients are very small, indicative of the fact that the model properly incorporates each of the given variables. As a final indicator of the quality of the model, SAS provides a classification table, replicated on the following page. The table is fairly self-explanatory, comparing the numbers of satellites for which the model predicts a .5 or higher probabilty of detection, and those that are actually detected. Conversely, numbers of satellites with a low probability of detection and those that are not actually detected are compared. As can be seen, this rough indicator of model performance indicates a successful prediction rate of 73.5%.

CLASSIFICATION TABLE

		Predicted		
		Negative	Positive	Total
True	Negative	19862	3499	23361
	Positive	5270	4444	9714
	TOTAL	25132	7943	33075

B. PROBABILITY OF DETECTION EXAMPLE

A computational example is now given in order to demonstrate how the fitted logistic regression model can be used to estimate NAVSPASUR detection performance. Suppose it is desired to calculate the probability of detecting a new threat satellite with the following parameters at fence penetration:

RCS = 1 m^2

INCLINATION = 60 degrees = 1.047 radians

ALTITUDE = 2000 nautical miles

LONGITUDE = 90 degrees west

These satellite parameters may be entered into the fitted g(x) equation, giving:

$$g(x) = -19.99357495 + 0.00100492(2000) - 0.00000024(2000)^2 + 0.39842267(1) + 0.38942569(90) - 0.00196066(90)^2 - 0.17300374(1.047)$$

$$= 0.440439$$

The probability the NAVSPASUR fence would detect this particular satellite pass is obtained by entering the calculated g(x) value into the inverse logit equation, as below:

$$\pi(x) = \frac{\exp[g(x)]}{1 + \exp[g(x)]}$$

$$= \frac{\exp[.440439]}{1 + \exp[.440439]}$$

$$= .6084$$

Thus the model gives a predicted probability of approximately .61 that a satellite with the characteristics listed above will be detected in a single pass through the NAVSPASUR fence. A particularly encouraging note about this result is that the exploratory GRAFSTAT analysis portion of this study yielded a similar, appoximately .60, probability of detection for a satellite with similar detection parameters. Figure 20 is an illustration of the estimated probability of detection for a satellite with the radar cross section and inclination of the example satellite, calculated over a range of fence crossing longitudes and altitudes.

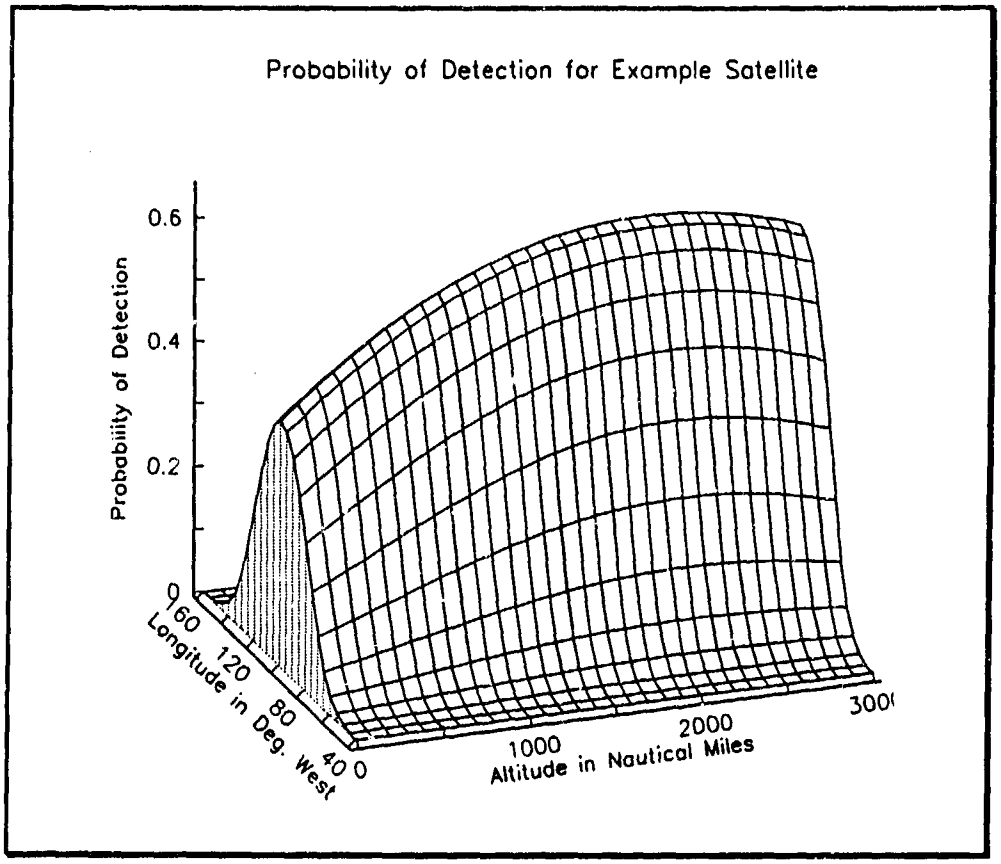

Figure 20. Logistic Regression Model Surface

VI. CONCLUSIONS AND AREAS FOR FURTHER STUDY

From the commencement of this study it was evident that the voluminous amount of data provided by NAVSPASUR would have to be handled in an efficient and highly automated fashion. The GRAFSTAT and SAS programs proved highly effective in meeting this task once the data set had been properly formatted and stored on the NPS Mainframe Computer. GRAFSTAT provided concise graphics, clearly illustrating important trends in the data. This was instrumental in providing the background necessary to then develop our logistic regression model with the SAS program. Several indicators point to the accuracy of our fitted probability of detection model. Both our graphical and logistic regression results correlate closely with each other as well as with the previous work completed by Mr. Robin Smith of NAVSPASUR. Also, it is apparent from the coefficients and general behavior of our model that the logistic regression model closely agrees with the physical theory involved with the fence. For example, from examining the model it is apparent that positive factors such as increasing a satellites' radar cross section will significantly increase its probability of detection, a point illustrated in the theory section of chapter II.

Some areas of this research merit further research. Analysis of additional observation periods would certainly be desirable in order to insure that 24 April 1991 was not a day of performance anomaly on the part of the fence. Additionally, researchers with a strong background in software design might provide an automated system for use by NAVSPASUR to continually update its' level of detection performance. Finally, additional study of this nature should be conducted upon completion of planned fence improvements such as the out of plane station program and software modifications.

APPENDIX A. SUMMARY STATISTICS

The following tables consist of summary statistics with regards to the data density diagrams of Chapter IV. Of particular note in the tables is the fact that the number of data elements varies slightly for altitude and radar cross section. An altitude gate was set at 3000 nautical miles in the data exploration section. It can be seen that this restriction caused minimal (less than 1500) reduction in the number of elements. A similar tactic was employed with radar cross section. As discussed in the main body of the thesis, these decisions were made in order to minimize the model degrading affect of far outlying data points.

I. Summary Statistics for Altitude

No. of elements	35753
Mean	1164
Std. deviation	567.28
Skewness	2.6784
Kurtosis	13.824
5^{th}-Percentile	575.96
95^{th}-percentile	1963.3
Median	997.6

II. Summary Statistics for Longitude

No. of elements	37181
Mean	99.268
Std. deviation	28.903
Skewness	0.0160
Kurtosis	2.119
5^{th}-Percentile	55.71
95^{th}-percentile	142.98
Median	99.4

III. Summary Statistics for Radar Cross Section

No. of elements	36866
Mean	1.449
Std. deviation	3.013
Skewness	2.869
Kurtosis	11.561
5^{th}-Percentile	0.003
95^{th}-percentile	8.700
Median	0.113

APPENDIX B. APL CODING

As briefly discussed earlier in the thesis, some new routines were constructed with Prof. Peter Lewis to enabie Grafstat to properly manipulate the NAVSPASUR data base. These various functions are coded in APL (A Programming Language), and are used to edit, and manipulate data desired when analyzing particular detection parameters. The ROWSTRIP routine was written to allow the stripping of satellite events with the specified characteristics from the data base. For example, this routine was used to select satellite records with certain cross sectional dimensions. Another routine, NCTABSLL, was used to tabularize groups of satellite events. For example, this routine can be used to display the observed number of events with altitudes between 100 and 3100 nautical miles, in 30 bins of 100 miles each. Finally, the routine PROBMATRIX, performs an approximate logistic regression with certain independent variables. Coding sequences for selected GRAFSTAT functions are shown below.

A. SATELLITES OF 1 M^2 RCS

In this short coding sequence SEL1 becomes the matrix of 1 m^2 satellites that are detected. SEL1B becomes those that are only predicted (detected or not detected). Detect/ No Detect information is contained in fields 4 through 9, one field for each NAVSPASUR receiver. Q21 and Q31 become the altitudes and longitudes of those satellites, respectively, that are detected. Q21B and Q31B are the same values for all satellites in the radar cross section group that are predicted to be seen. TAB1 and TAB1B enter numbers of detections and predictions into the appropriate bin. PROBMATRIX then utilizes all of this sorted data to calculate a probability of detection for each respective bin. Similar coding sequences follow for each of the individual receiver sites.

```
SEL1<-(Q1 ≤ =1.5) & (Q1=0.5) & (Q4 thru Q9 are at least = 2)
SEL1B<-(Q1 ≤ =1.5) & (Q=0.5)
Q21<-Q2 ROWSTRIP SEL1
Q31<-Q3 ROWSTRIP SEL1
Q21B<-Q2 ROWSTRIP SEL1B
Q31B<-Q3 ROWSTRIP SEL1B
TAB1<- (60 160 20) (100 3100 30) NCTABSLL Q31 Q21
TAB1B<- (60 160 20) (100 3100 30)NCTABSLL Q31B Q21B
PROB1<-TAB1B PROBMATIX  TAB1
```

B. SATELLITES OF 1 M^2 RCS: INDIVIDUAL STATIONS

1. San Diego

In this coding sequence and those that follow, The Detect/No Detect results of only the station in question are queried. As before, SEL1SD becomes the matrix of 1 m^2 satellites that are detected. SEL1SDB becomes those that are only predicted (detected or not detected). Detect/ No Detect information is contained in field 4 for the San Diego receiver. Q21SD and Q31SD become the altitudes and longitudes of those satellites, respectively, that are detected. Q21SDB and Q31SDB are the same values for all satellites in the radar cross section group that are predicted to be seen. TAB1SD and TAB1SDB enter numbers of detections and predictions into the appropriate bin. PROBMATRIX then utilizes all of this sorted data to calculate a probability of detection for each respective bin.

```
SEL1SD<-(Q1 ≤ =1.5) & (Q1=0.5) & (Q4 at least = 2)
SEL1SDB<-(Q1 ≤ =1.5) & (Q=0.5) & (Q4 at least = 1)
Q21SD<-Q2 ROWSTRIP SEL1SD
Q31SD<-Q3 ROWSTRIP SEL1SD
Q21SDB<-Q2 ROWSTRIP SEL1SDB
Q31SDB<-Q3 ROWSTRIP SEL1SDB
TAB1SD<- (60 160 20) (100 3100 30) NCTABSLL Q31SD Q21SD
TAB1SDB<- (60 160 20) (100 3100 30) NCTABSLL Q31SDB Q21SDB
PROB1SD<-TAB1SDB PROBMATIX  TAB1SD
```

2. **Red River**

```
SEL1RR<-(Q1 ≤ =1.5) & (Q1=0.5) & (Q6 at least = 2)
SEL1RRB<-(Q1 ≤ =1.5) & (Q=0.5) & (Q6 at least = 1)
Q21RR<-Q2 ROWSTRIP SEL1RR
Q31RR<-Q3 ROWSTRIP SEL1RR
Q21RRB<-Q2 ROWSTRIP SEL1RRB
Q31RRB<-Q3 ROWSTRIP SEL1RRB
TAB1RR<- (60 160 20) (100 3100 30) NCTABSLL Q31RR Q21RR
TAB1RRB<- (60 160 20) (100 3100 30)NCTABSLL Q31RRB Q21RRB
PROB1RR<-TAB1RRB PROBMATIX  TAB1RR
```

3. **Silver Lake**

SEL1SL<-(Q1 ≤ =1.5) & (Q1=0.5) & (Q7 at least = 2)

SEL1SLB<-(Q1 ≤ =1.5) & (Q=0.5) & (Q7 at least = 1)

Q21SL<-Q2 ROWSTRIP SEL1SL

Q31SL<-Q3 ROWSTRIP SEL1SL

Q21SLB<-Q2 ROWSTRIP SEL1SLB

Q31SLB<-Q3 ROWSTRIP SEL1SLB

TAB1SL<- (60 160 20) (100 3100 30) NCTABSLL Q31SL Q21SL

TAB1SLB<- (60 160 20) (100 3100 30)NCTABSLL Q31SLB Q21SLB

PROB1SL<-TAB1SLB PROBMATIX   TAB1SL

4. **Hawkinsville**

SEL1H<-(Q1 ≤ =1.5) & (Q1=0.5) & (Q8 at least = 2)

SEL1HB<-(Q1 ≤ =1.5) & (Q=0.5) & (Q8 at least = 1)

Q21H<-Q2 ROWSTRIP SEL1H

Q31<-Q3 ROWSTRIP SEL1H

Q21HB<-Q2 ROWSTRIP SEL1HB

Q31HB<-Q3 ROWSTRIP SEL1HB

TAB1H<- (60 160 20) (100 3100 30) NCTABSLL Q31H Q21H

TAB1HB<- (60 160 20) (100 3100 30) NCTABSLL Q31HB Q21HB

PROB1H<-TAB1HB PROBMATIX   TAB1H

5. **Tattnal**

SEL1T<-(Q1 ≤ =1.5) & (Q1=0.5) & (Q9 at least = 2)

SEL1TB<-(Q1 ≤ =1.5) & (Q=0.5) & (Q9 at least = 1)

```
Q21T<-Q2 ROWSTRIP SEL1T
Q31T<-Q3 ROWSTRIP SEL1T
Q21TB<-Q2 ROWSTRIP SEL1TB
Q31TB<-Q3 ROWSTRIP SEL1TB
TAB1T<- (60 160 20) (100 3100 30) NCTABSLL Q31T Q21T
TAB1TB<- (60 160 20) (100 3100 30) NCTABSLL Q31TB Q21TB
PROB1T<-TAB1TB PROBMATIX  TAB1T
```

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